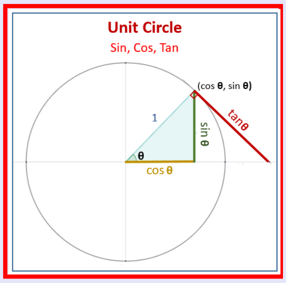
Math 241

Winter 2023

Lecture 1



Math 241S

5 Weeks

M-Th

7:00 - 11:40

Important:

1) Arrive on time and stay for the entire time.

2) Access to Canvas. Download Canvas for student App.

3) Have a scientific calculator with you in class at all time.

4) Final Exam: February 2, 2023.

Some Review:

1) Solve: $2(x-3) = x+5$ → Solution Set

$2x - 6 = x + 5$

$2x - x = 5 + 6$ → $x = 11$

1) $\{11\}$

2) Simplify: $(x-5)^2 + 10x$

$= (x-5)(x-5) + 10x$

$= x^2 - 5x - 5x + 25 + 10x$

$= x^2 \cancel{-10x} + 25 \cancel{+10x}$

$= \boxed{x^2 + 25}$

2) $x^2 + 25$

3) Factor Completely:

a) $x^2 + 6x = x(x+6)$

a) $x(x+6)$

b) $x^2 + 6x + 9$ 　 1,9 　 $\boxed{3,3}$

L.C. $= 1$

$= (x+3)(x+3)$

$= (x+3)^2$

b) $(x+3)^2$

c) $x^2 - x - 30$ 　 1,30 　 2,15 　 3,10 　 $\boxed{5,6}$

$= (x+5)(x-6)$

c) $(x+5)(x-6)$

LC $= 4$

d) $4x^2 + 5x - 9 = \underbrace{4x^2 - 4x} + \underbrace{9x - 9}$

Product $= -36$

Sum $= 5$

-36: $-1, 36$ 　 $-2, 18$ 　 $-3, 12$ 　 $\boxed{-4, 9}$ 　 $-6, 6$

$= 4x(x-1) + 9(x-1)$

$= \boxed{(x-1)(4x+9)}$

c) $(4x+9)(x-1)$

Use quadratic formula to solve $2x^2 - 3x - 5 = 0$

Quadratic Equation $ax^2 + bx + c = 0,\ a \neq 0$

Quadratic Formula $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a = 2,\ b = -3,\ c = -5$

$b^2 - 4ac = (-3)^2 - 4(2)(-5)$ Discriminant

$= 9 + 40 = 49$

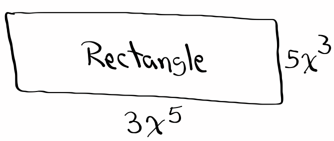
$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-(-3) \pm \sqrt{49}}{2(2)} = \dfrac{3 \pm 7}{4}$

$x = \dfrac{3+7}{4} = \dfrac{10}{4} = \dfrac{5}{2}$

$x = \dfrac{3-7}{4} = \dfrac{-4}{4} = -1$

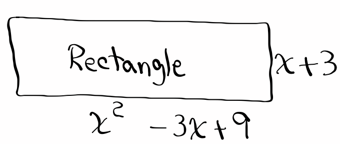
$\Rightarrow$ Solution Set $\left\{-1, \dfrac{5}{2}\right\}$

Find Area & Perimeter:

1) 

$A = LW$
$= 3x^5 \cdot 5x^3$
$= 15x^{5+3} = 15x^8$

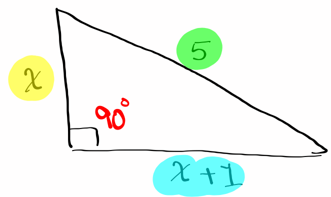
$P = 2L + 2W$
$= 2(3x^5) + 2(5x^3)$
$= \boxed{6x^5 + 10x^3}$

2) 

$A = LW$
$= (x+3)(x^2 - 3x + 9)$
$= x^3 \cancel{-3x^2} \cancel{+9x} \cancel{+3x^2} \cancel{-9x} + 27$
$= \boxed{x^3 + 27}$

$P = 2L + 2W$
$= 2(x^2 - 3x + 9) + 2(x+3)$
$= 2x^2 - 6x + 18 + 2x + 6 = \boxed{2x^2 - 4x + 24}$

Find x:



Right Triangle

Pythagorean Thrm

$a^2 + b^2 = c^2$

$x^2 + (x+1)^2 = 5^2$

$x^2 + (x+1)(x+1) = 25$

$x^2 + x^2 + x + x + 1 - 25 = 0$

$2x^2 + 2x - 24 = 0$

Divide by 2

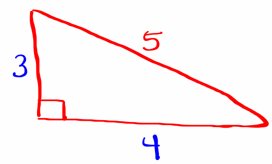
$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

Zero-Product Rule

$x+4=0$ OR $x-3=0$

$\cancel{x=-4}$ $\boxed{x=3}$



Solve and graph

$2x + 7 > 4x - 9$

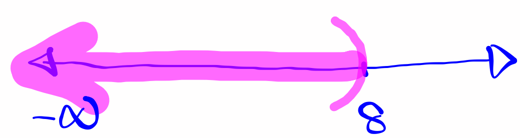
$2x - 4x > -9 - 7$

$-2x > -16$

Divide by -2

$\frac{-2}{-2}x < \frac{-16}{-2}$

$\boxed{x < 8}$



Set-Builder notation

$\{x \mid x < 8\}$

Interval notation

$(-\infty, 8)$

Solve and graph

$-5 < -2x + 1 \le 9$

Hint: Isolate x in the middle.

$-5 - 1 < -2x + 1 - 1 \le 9 - 1$

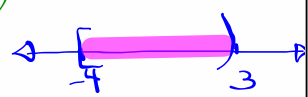
$-6 < -2x \le 8$

Divide by -2

$\frac{-6}{-2} > \frac{-2}{-2}x \ge \frac{8}{-2}$

$3 > x \ge -4$

$\boxed{-4 \le x < 3}$



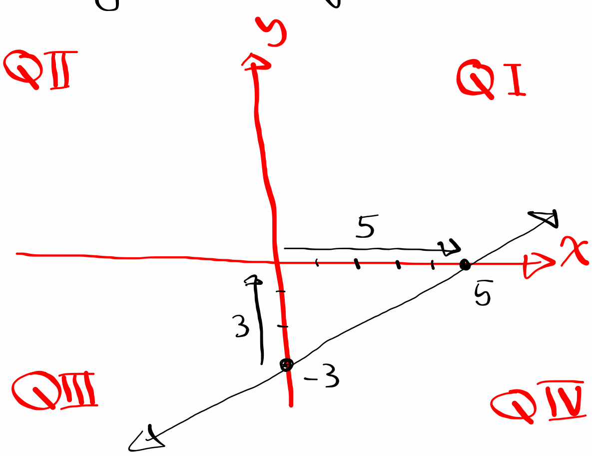
S.B.N. $\{x \mid -4 \le x < 3\}$ → Set-Builder-Notation

I.N. $[-4, 3)$ → Interval Notation

Graph $3x - 5y = 15$ by completing the chart below

x	y
0	-3
5	0

Intercept Method



Increasing

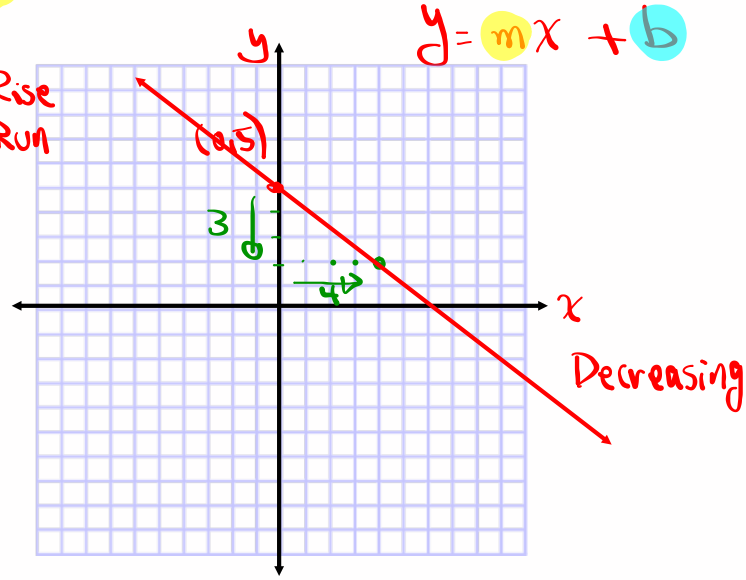
Ratio $= \frac{\text{Rise}}{\text{Run}} = \frac{3}{5}$ Slope

Graph $y = \frac{-3}{4}x + 5$ using Slope-Int. Form

$y = mx + b$

Slope $m = \frac{-3}{4}$ ← Rise / ← Run

Y-Int $(0, 5)$



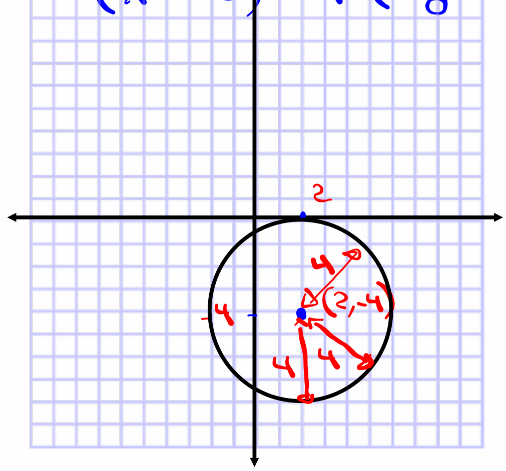
Equation of a Circle: Center (h, k)

$(x - h)^2 + (y - k)^2 = r^2$ Radius r

Ex:

$(x - 2)^2 + (y + 4)^2 = 16$ Center $(2, -4)$

Radius $r = 4$



Draw

$$x^2 + (y-5)^2 = 25$$

Center $(0,5)$

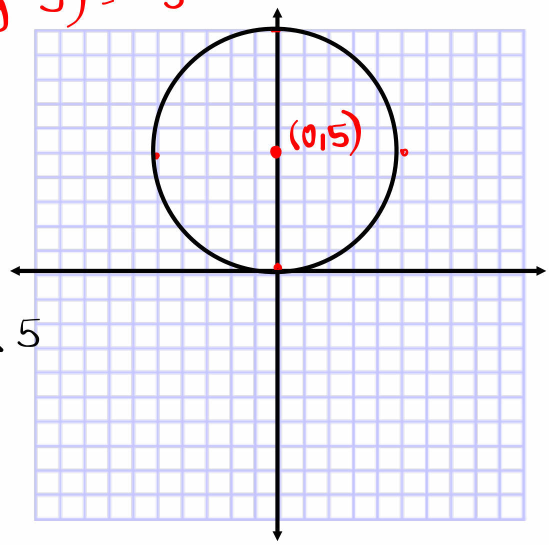
Radius 5

$$(x-0)^2 + (y-5)^2 = 5^2$$

SG 1

Due: 11:59 PM Jan. 4

Available: 11:59 PM Jan. 5



Distance formula between two points:

$A(x_1, y_1)$, $B(x_2, y_2)$

$$d(A,B) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Find the distance from $\boxed{(-2,3)}$ (A) to $\boxed{(5,-4)}$ (B)

$$d(A,B) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(-2 - 5)^2 + (3 - -4)^2}$$

$$= \sqrt{(-7)^2 + (7)^2} = \sqrt{49+49} = \sqrt{98} \approx 10$$

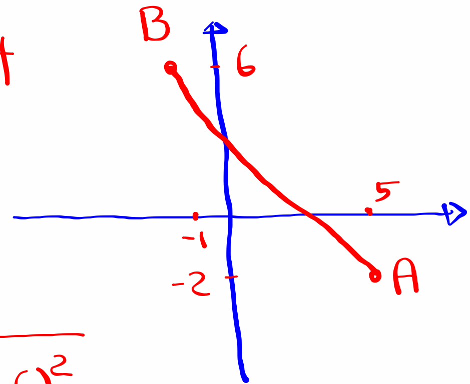
$$= \sqrt{49 \cdot 2} = \sqrt{49} \cdot \sqrt{2} = \boxed{7\sqrt{2}}$$

$A(5,-2)$, $B(-1,6)$

1) Plot A & B

2) Draw $\overline{AB}$ line segment

3) Find $d(A,B)$



$$d(A,B) = \sqrt{(5--1)^2 + (-2-6)^2}$$

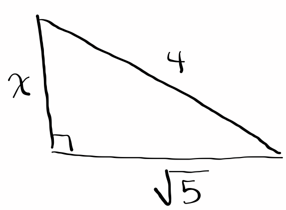
$$= \sqrt{6^2 + (-8)^2} = \sqrt{36+64} = \sqrt{100} = \boxed{10}$$

Simplify

$$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{-1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = \boxed{1}$$

Find x:



Right Triangle

Pythagorean thrm

$$x^2 + (\sqrt{5})^2 = 4^2$$

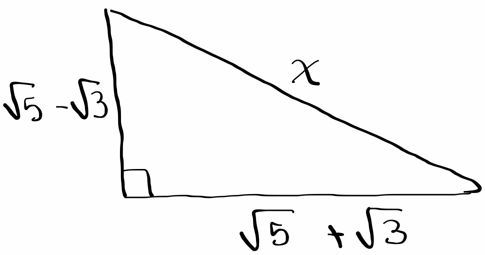
$$x^2 + 5 = 16$$

$$x^2 = 11 \qquad \boxed{x = \sqrt{11}}$$

Simplify

$$\left(\frac{\sqrt{11}}{4}\right)^2 + \left(\frac{\sqrt{5}}{4}\right)^2 = \frac{11}{16} + \frac{5}{16} = \frac{16}{16} = \boxed{1}$$

Find x:



Using Pythagorean thrm

$$x^2 = (\sqrt{5}-\sqrt{3})^2 + (\sqrt{5}+\sqrt{3})^2$$

$$= (\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3}) + (\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{3})$$

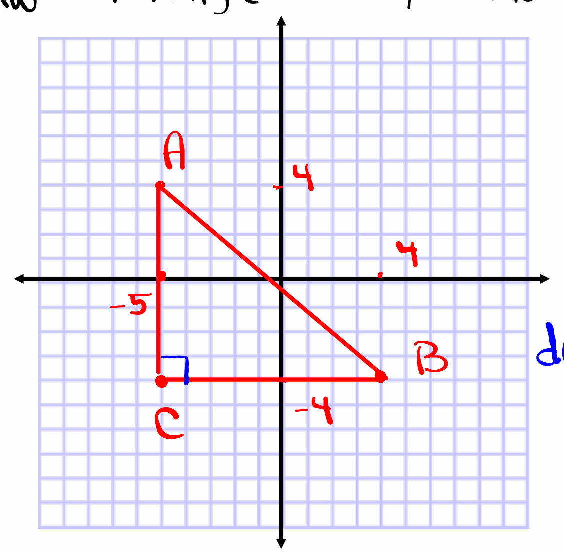
$$x^2 = \sqrt{25} - \cancel{\sqrt{15}} - \cancel{\sqrt{15}} + \sqrt{9} + \sqrt{25} + \cancel{\sqrt{15}} + \cancel{\sqrt{15}} + \sqrt{9}$$

$$= 5 + 3 + 5 + 3 = 16 \qquad x^2 = 16 \qquad x = \sqrt{16}$$

$$\boxed{x = 4}$$

A(−5, 4), B(4, −4), C(−5, 4)

Draw triangle ABC, find its area & perimeter



$\overline{AC} \to 8$

$\overline{BC} \to 9$

$$\text{Area} = \frac{bh}{2} = \frac{8 \cdot 9}{2} = \boxed{36}$$

$$d(A,B) = \sqrt{(-5-4)^2 + (4-(-4))^2}$$

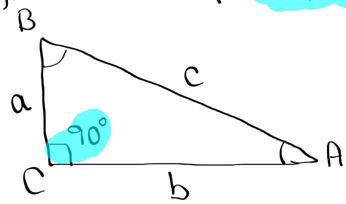
$$= \sqrt{(-9)^2 + 8^2}$$

$$= \sqrt{81 + 64} = \sqrt{145}$$

$$P = a + b + c = 8 + 9 + \sqrt{145} = \boxed{17 + \sqrt{145}} \approx 17 + 12 \approx \boxed{29}$$

What is Trigonometry?

It is a relationship between sides and angles in any right triangle.



Using Pythagorean thrm

$$a^2 + b^2 = c^2$$

$$\angle A + \angle B = 90^\circ$$

The Followings are trig. Functions:

Sine → Sin

Cosine → Cos

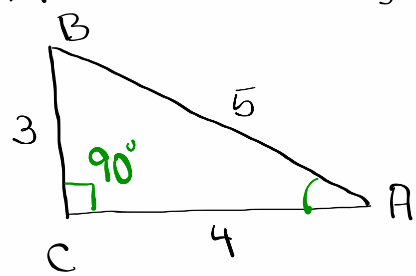
tangent → tan

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{b}{c}$$

$$\tan A = \frac{\text{opposite}}{\text{Adjacent}} = \frac{a}{b}$$

Consider the triangle below



If we verify the Pythagorean Thrm $\Rightarrow \angle C = 90^\circ$

$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

$$25 = 25 \checkmark$$

$$\sin A = \frac{3}{5}$$

$$\cos A = \frac{4}{5}$$

$$\tan A = \frac{3}{4}$$

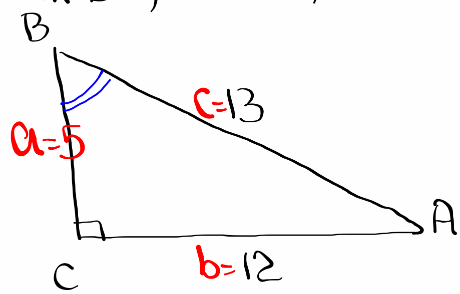
Verify that

$$\boxed{\sin^2 A + \cos^2 A = 1}$$

$$\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{9}{25} + \frac{16}{25} = \frac{25}{25}$$

$$= 1$$

Find the missing side, then find $\sin B$, $\cos B$, and $\tan B$.



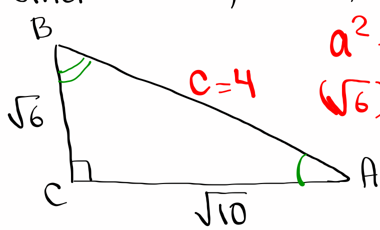
$$a^2 + b^2 = c^2$$
$$a^2 + 12^2 = 13^2$$
$$a^2 + 144 = 169$$
$$a^2 = 25$$
$$\boxed{a = 5}$$

$$\sin B = \frac{\text{Opp.}}{\text{Hyp.}} = \frac{12}{13} \qquad \cos B = \frac{\text{Adj.}}{\text{Hyp.}} = \frac{5}{13}$$

$$\tan B = \frac{\text{Opp.}}{\text{Adj.}} = \frac{12}{5}$$

Avoid mixed-numbers & Decimals.

Find $\sin A$, $\cos B$, $\tan A$.



$$a^2 + b^2 = c^2$$
$$(\sqrt{6})^2 + (\sqrt{10})^2 = c^2 \rightarrow c^2 = 16 \rightarrow \boxed{c = 4}$$

$$\sin A = \frac{\text{Opp.}}{\text{Hyp.}} = \frac{\sqrt{6}}{4}$$

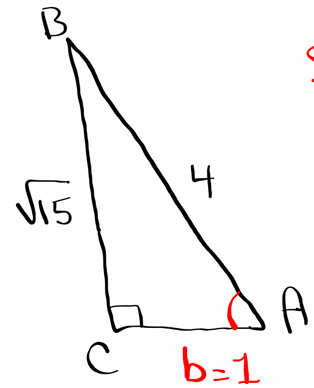
$$\tan A = \frac{\text{Opp.}}{\text{Adj.}} = \frac{\sqrt{6}}{\sqrt{10}} \qquad \cos B = \frac{\text{adj.}}{\text{Hyp.}} = \frac{\sqrt{6}}{4}$$

$$= \frac{\cancel{\sqrt{2}}\sqrt{3}}{\cancel{\sqrt{2}}\sqrt{5}} = \frac{\sqrt{3} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{15}}{\sqrt{25}} = \boxed{\frac{\sqrt{15}}{5}}$$

This answer must be rationalized.

"No radicals in the denominator"

Find Sin A , Cos A, and tan A



$\sin A = \frac{\text{opp.}}{\text{Hyp.}} = \frac{\sqrt{15}}{4}$

$\cos A = \frac{\text{Adj.}}{\text{Hyp.}} = \frac{1}{4}$

$a^2 + b^2 = c^2$

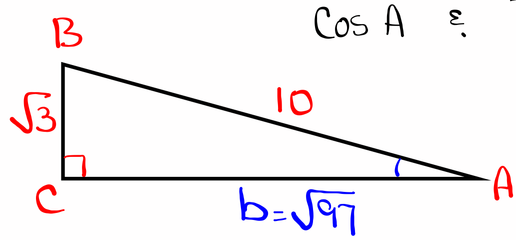
$(\sqrt{15})^2 + b^2 = 4^2 \rightarrow \boxed{b=1}$

$\tan A = \frac{\text{opp.}}{\text{adj.}} = \frac{\sqrt{15}}{1} = \boxed{\sqrt{15}}$

Consider triangle ABC with C=90° and

$\sin A = \frac{\sqrt{3}}{10}$. Find missing side and

Cos A & tan A.



$\sin A = \frac{\sqrt{3}}{10}$

opp. $\sqrt{3}$

Hyp. 10

$(\sqrt{3})^2 + b^2 = 10^2$

$3 + b^2 = 100 \rightarrow b = \sqrt{97}$

$\cos A = \frac{\sqrt{97}}{10}$

$\tan A = \frac{\sqrt{3}}{\sqrt{97}} \cdot \frac{\sqrt{97}}{\sqrt{97}}$

$= \frac{\sqrt{291}}{97}$

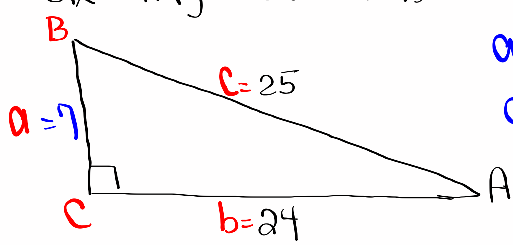
Now reciprocal Trig. functions:

Cosecant → Csc → $\csc A = \dfrac{1}{\sin A}$

Secant → Sec → $\sec A = \dfrac{1}{\cos A}$

Cotangent → Cot → $\cot A = \dfrac{1}{\tan A}$

Find the missing side, then find all six trig. functions for angle A.



$a^2 + b^2 = c^2$

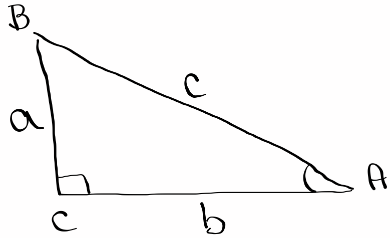
$a^2 + 24^2 = 25^2$

$\boxed{a = 7}$

$\sin A = \dfrac{7}{25}$	$\csc A = \dfrac{25}{7}$
$\cos A = \dfrac{24}{25}$	$\sec A = \dfrac{25}{24}$
$\tan A = \dfrac{7}{24}$	$\cot A = \dfrac{24}{7}$

Consider the triangle below, verify that

$$1+\tan^2 A = \sec^2 A$$



$\tan A = \dfrac{a}{b}$

$\cos A = \dfrac{b}{c}$, $\sec A = \dfrac{c}{b}$

$a^2+b^2=c^2$

$$1+\left(\frac{a}{b}\right)^2=\left(\frac{c}{b}\right)^2$$

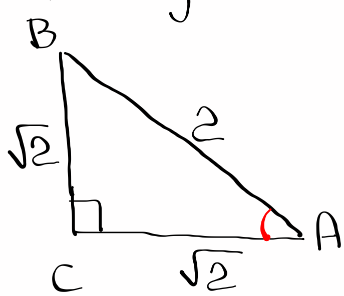
$$\boxed{1}+\frac{a^2}{b^2}=\frac{c^2}{b^2}$$

$$\frac{b^2}{b^2}+\frac{a^2}{b^2}=\frac{c^2}{b^2}$$

$$\frac{b^2+a^2}{b^2}=\frac{c^2}{b^2}$$

$$\frac{c^2}{b^2}=\frac{c^2}{b^2}\ \checkmark$$

Use the right triangle below to find all six trig. functions of angle A.



Let's verify the Pythagorean thrm $(\sqrt{2})^2+(\sqrt{2})^2=2^2$

$$2+2=4\ \checkmark$$

$\sin A = \dfrac{\sqrt{2}}{2}$

$\csc A = \dfrac{2}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\cancel{2}\sqrt{2}}{\cancel{\sqrt{4}}} = \sqrt{2}$

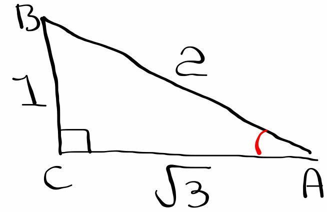
$\cos A = \dfrac{\sqrt{2}}{2}$

$\sec A = \dfrac{2}{\sqrt{2}} = \text{- - - - -} = \sqrt{2}$

$\tan A = \dfrac{\sqrt{2}}{\sqrt{2}} = 1$

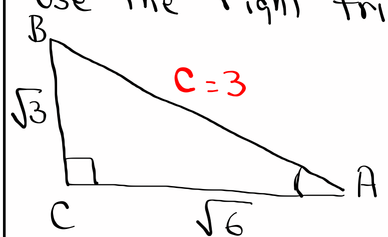
$\cot A = 1$

Complete the chart below using



$\sin A = \frac{1}{2}$	$\csc A = 2$
$\cos A = \frac{\sqrt{3}}{2}$	$\sec A = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
$\tan A = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	$\cot A = \sqrt{3}$

Use the right triangle below to complete this chart.



$\sin A = \frac{\sqrt{3}}{3}$	$\csc A = \sqrt{3}$
$\cos A = \frac{\sqrt{6}}{3}$	$\sec A = \frac{\sqrt{6}}{2}$
$\tan A = \frac{\sqrt{2}}{2}$	$\cot A = \sqrt{2}$

$(\sqrt{3})^2 + (\sqrt{6})^2 = c^2$

$3 + 6 = c^2$

$c^2 = 9$

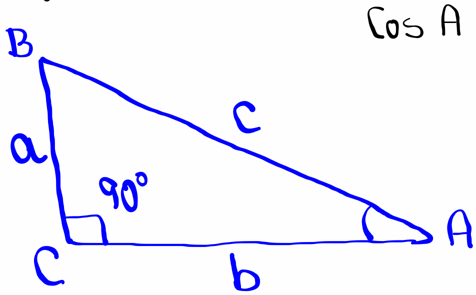
$\boxed{c = 3}$

$\tan A = \frac{\sqrt{3}}{\sqrt{6}} = \frac{\cancel{\sqrt{3}} \cdot 1}{\cancel{\sqrt{3}} \cdot \sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\csc A = \frac{3 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{3\sqrt{3}}{\sqrt{9}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

$\sec A = \frac{3 \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}} = \frac{\cancel{3}\sqrt{6}}{\cancel{6}\,2}$

Prove $\tan A = \dfrac{\sin A}{\cos A}$



$\sin A = \dfrac{a}{c}$

$\cos A = \dfrac{b}{c}$

$$\tan A = \frac{a}{b} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{\sin A}{\cos A} \checkmark$$

Divide RHS by C, Top & bottom

Simplify

$$(\sin A + \cos A)^2 + (\sin A - \cos A)^2$$

$$= (\sin A + \cos A)(\sin A + \cos A) +$$

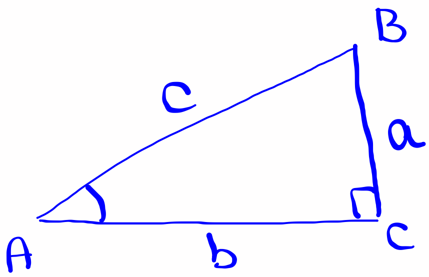
$$(\sin A - \cos A)(\sin A - \cos A)$$

$$= \sin^2 A + \cancel{\sin A \cos A} + \cancel{\sin A \cos A} + \cos^2 A +$$

$$\sin^2 A - \cancel{\sin A \cos A} - \cancel{\sin A \cos A} + \cos^2 A$$

$$= 1 + 1 = \boxed{2}$$

Prove $\sin^2 A + \cos^2 A = 1$



we know $a^2 + b^2 = c^2$

we also know

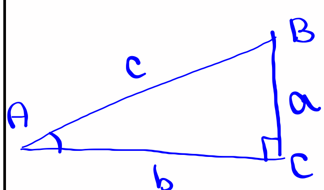
$\sin A = \frac{a}{c}$, $\cos A = \frac{b}{c}$

$$\sin^2 A + \cos^2 A = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$$

$$= \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2 + b^2}{c^2}$$

$$= \frac{c^2}{c^2} = \boxed{1}$$

Prove $1 + \cot^2 A = \csc^2 A$



$\cot A = \frac{b}{a}$

$\tan A = \frac{a}{b}$

$\csc A = \frac{c}{a}$

$\sin A = \frac{a}{c}$

$$1 + \cot^2 A = 1 + \left(\frac{b}{a}\right)^2 = \boxed{1} + \frac{b^2}{a^2} = \boxed{\frac{a^2}{a^2}} + \frac{b^2}{a^2}$$

$1 + \cot^2 A = \csc^2 A$

$$= \frac{a^2 + b^2}{a^2}$$

$$= \frac{c^2}{a^2}$$

$$= \left(\frac{c}{a}\right)^2$$

$$= \csc^2 A \checkmark$$

Class QZ 1:

Solve $3x^2 - 4x - 7 = 0$ by Quadratic Formula.

$a = 3$, $b = -4$, $c = -7$

$b^2 - 4ac = (-4)^2 - 4(3)(-7) = 16 + 84 = 100$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{100}}{2(3)} = \frac{4 \pm 10}{6}$$

$$x = \frac{4+10}{6} = \frac{14}{6} = \boxed{\frac{7}{3}} \qquad x = \frac{4-10}{6} = \frac{-6}{6} = \boxed{-1}$$

$$\boxed{\left\{-1, \frac{7}{3}\right\}}$$